UNEMPLOYMENT PERSISTENCE IN ITALY. AN ECONOMETRIC ANALYSIS WITH MULTIVARIATE TIME VARYING PARAMETER MODELS [1]

Gianni Amisano[2], Massimiliano Serati[3]

1. Introduction

This paper aims at investigating the causes of unemployment persistence in Italy, by means of aggregate quarterly data. The model that we use as reference, as explained in detail in section 2, is very simple, but we believe that it can be validly used to explain what happened in the Italian labour market during the last 20 years. In a nutshell, the model consists of a real wage formation mechanism based on *insider-outsider* considerations, of a mark-up pricing rule justified by the assumption of monopolistic competition on the product market, of a labour supply function, according to which participation depends on real salary expectations and on workers' marginalisation dynamics. The model is completed by a labour demand function, obtained by jointly considering an aggregate production function and an aggregate demand function.

In the sample period being considered (1975-1995), very deep structural modifications have affected the Italian economy, and section (3) contains an account of these. Whenever it is not possible to explicitly model these phenomena, it is necessary to conjugate the theoretical model with econometric techniques capable of modelling structural instability phenomena.

From the methodological point of view, we estimate a reduced form dynamic model, in the form of an unrestricted time varying parameter *VAR* model with exogenous variables, as in the *BVAR* methodology (see [14]).

The paper is organised as follows. Section 2 describes the theoretical model we us as a reference framework, putting special emphasis on its alternative representations, dynamic properties, and long run equilibrium solution. Section 3 contains a detailed account of the main sources of structural change that have hit the Italian labour market, and that in our view justify the adoption of a time varying parameter framework. Section 4 contains the justification of why we decided to estimate a reduced form model. Section 5 describes the Bayesian inferential techniques that we use, and section 6 contains the results of the estimated model. Section 7 deals with a new proposal for dealing with parameter time variability that we consider as particularly appropriate for the sample period that we analyse, and section 8 is devoted to the conclusions and the lines of future research.

2. The Model

2.1 Overall structure

The theoretical framework underlying our analysis is our modification (see [1]) of the model proposed by Layard, Nickell and Jackman (henceforth LNJ, see [21]). It is a microfounded structural model describing a typical economy in which a large number of firms sets prices and output in order to maximize expected profits. Also wage setting does not reflect perfect competition in the labour market, since the wage depends both on insider and outsider factors like intensity and duration of unemployment benefits; a significant role in wage definition is also played by exogenous components like, *in primis,* labour taxation and unions activity. It is just because it allows both for aggregate demand factors and also for different kinds of labour market rigidities that the LNJ model seems to be a useful tool in order to study the determinants of unemployment persistence, especially for European economies.

A stylised summary of the logical scheme of the model can be presented as follows: given the capital stock, the optimal prices are set by each firm on the basis of expectations on market demand and costs; prices drive wages, whereas output is determined in order to match the share of market demand faced by the single firm. Finally, the employment level is set to produce planned output.

In the light of its equilibrium features, the system of equations from (2.1) to (2.6) can be classified as a NAIRU model; in fact, only supply side factors can influence the long run employment and output levels, whereas demand factors and policy measures can play a significant role only in the short run.

One of the most limiting features of the original LNJ model is the incompleteness of the labour market description it provides. No consideration, in fact, is made on the supply side and the labour force is handled as a fully exogenous variable.

In order to overcome this problem and to split the unemployment behaviour in its supply and demand components, we introduce a substantial change into the model: we add to its original structure a participation equation in which the behaviour of the labour force is supposed to depend (positively) on the expected real wage and (negatively) on the past level of unemployment and on the labour taxes level; the supposed negative trade off between participation and past unemployment is intended to model a discouraged workers effect.

In our version the model is characterized by five endogenous variables[4] : output (*y*), prices (*p*), wages (*w*), employment (*n*) and labour force (*l*). The exogenous variables are the capital stock (*k*), a fiscal stance indicator (*x*), the money stock (*m*), an index of duration and intensity of the unemployment benefits (*UB*) and a measure of labour taxes (*LT*). The whole structure is presented in equations from (2.1) to (2.6):

$$y_t = \mathrm{a}n_t + (1-\mathrm{a})k_t + \mathrm{e}_t,\ \varepsilon_t = \frac{?^{\mathrm{e}}_t}{1-\rho_{\mathrm{e}}L},\ ?^{\mathrm{e}}_t \sim w.n.(0,\mathrm{s}_{\mathrm{e}}) \qquad (2.1)$$

$$y_t = \mathrm{s}x_t + ?(m_t - p_t) + ?_t,\ ?_t = \frac{?^?_t}{1-?_?L},\ ?^?_t \sim w.n.(0,\mathrm{s}_?) \qquad (2.2)$$

$$p_t = w_t + \text{ß}(y_t - k_t) + ?_t,\ ?_t = \frac{?^?_t}{1-?_?L},\ ?^?_t \sim w.n.(0,\mathrm{s}_?),\ \text{ß} = \frac{1\text{-}\mathrm{a}}{\mathrm{a}} \qquad (2.3)$$

$$w_t = p^e_t + \mathrm{a}\text{ß}k_t - \mathrm{a}\text{ß}?n_{t-1} - (1-?)\mathrm{a}\text{ß}l^e + \mathrm{k}UB_t + \mu LT_t + ?^?_t, \qquad (2.4)$$

$$?^?_t \sim w.n.(0,s_?)$$

$$l_t = ?n_{t-1} - ?l_{t-1} + ?(w-p)^e_t - ?LT_t + \mathrm{f}\,UB_t + ?^l_t,\ ?^l_t \sim w.n.(0,\mathrm{s}_l) \qquad (2.5)$$

$$u_t \equiv l_t - n_t \qquad (2.6)$$

where α and λ are non negative parameters smaller than one, γ,κ,μ,ξ,σ,ψ are expected to be non-negative, *L* is the lag operator and the superscript *e* represents expectation conditional on information available one period before.

Equation (2.1) describes the log-linear version of a production function with constant returns technology, equation (2.2) is the aggregate demand function obtained as the solution of a IS-LM system and (2.3) is the price setting equation.

The wage-setting equation (2.4) is slightly different from the original one contained in LNJ; the expected real wage rate is defined as the weighted sum of two components: an insider component for which level $n^e_t = n_{t-1}$, and an outsider one such that $n^e_t = l^e_t$:

$$w_t - p^e_t = ?(w_t - p^e_t : n^e_t = n_{t-1}) + (1-?)(w_t - p^e_t : n^e_t = l^e_t) \qquad (2.7)$$

Including unemployment benefits and adding to (2.7) a labour taxes measure[5], it is straightforward[6] to obtain expression (2.4).

In equation (2.5) the evolution of the labour force is endogenously defined in accordance with the scheme previously presented, whereas (2.6) is a simple identity defining unemployment.

One point again is worth noting: while LNJ give only a simultaneous representation of relationships among variables in equations (2.1), (2.2) and (2.3) we make them dynamic by modelling their innovations as generated by stationary[7] autoregressive processes of the first order.

2.2 Linear representation

Collecting the endogenous variables in the (5×1) vector $\boldsymbol{y_t}$ and the exogenous ones in the (5×1) vector $\boldsymbol{x_t}$, the matrix representation of system (2.1)-(2.6) is[8] :

$$\mathbf{A}_0\mathbf{y}_t - \mathbf{A}_1\mathbf{y}_{t-1} + \mathbf{G}\mathbf{y}_t^e + \mathbf{B}_0\mathbf{x}_t + \mathbf{B}_1\mathbf{x}_{t-1} = ?_t \tag{2.8}$$

Taking expectations of (2.8), we obtain:

$$\mathbf{y}_t^e = [\mathbf{A}_1^*\mathbf{y}_{t-1} + \mathbf{B}_0^*\mathbf{x}_t^e + \mathbf{B}_1^*\mathbf{x}_{t-1}], \tag{2.9}$$

$$\mathbf{A}_1^* = -(\mathbf{A}_0 + \mathbf{G})^{-1}\mathbf{A}_1, \mathbf{B}_0^* = -(\mathbf{A}_0 + \mathbf{G})^{-1}\mathbf{B}_0, \mathbf{B}_1^* = -(\mathbf{A}_0 + \mathbf{G})^{-1}\mathbf{B}_1 \tag{2.10}$$

Let us define[9] :

$$\mathbf{x}_t^e = \left[k_t \;\; x_{t\text{-}1} \;\; m_{t\text{-}1} \;\; UB_{t\text{-}1} \;\; LT_{t\text{-}1}\right] \tag{2.11}$$

and by substitution of (2.9) in (2.8) we can solve the model with respect to expectations:

$$\mathbf{A}_0\mathbf{y}_t + \mathbf{A}_1^{**}\mathbf{y}_{t-1} + \mathbf{B}_0^{**}\mathbf{x}_t + \mathbf{B}_1^{**}\mathbf{x}_{t-1} = ?_t \tag{2.12}$$

$$\mathbf{A}_1^{**} = \mathbf{G}\mathbf{A}_1^* + ?_1, ?_0^{**} = \mathbf{G}?_0^*\mathbf{D}_1 + ?_0,$$

$$\mathbf{B}_1^{**} = \mathbf{G}?_0^*\mathbf{D}_2 + \left[\mathbf{G}\mathbf{B}_1^* + \mathbf{B}_1\right] \tag{2.13}$$

$$\underset{(5\times5)}{\mathbf{D}_1} = \begin{bmatrix} 1 & \mathbf{0}' \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \mathbf{D}_2 = \mathbf{I}_5 - \mathbf{D}_1 = \begin{bmatrix} 0 & \mathbf{0}' \\ \mathbf{0} & \mathbf{I}_4 \end{bmatrix}$$

2.3 Equilibrium solution

Starting from (2.12), the static equilibrium solution of the model is obtained by setting $\mathbf{y}_t = \mathbf{y}_{t\text{-}1} = \mathbf{y}^*$ and $\mathbf{x}_t = \mathbf{x}_{t\text{-}1} = \mathbf{x}^*$ so that:

$$\mathbf{y} = -\left(\mathbf{A}_0 + \mathbf{A}_1^{**}\right)^{-1}\left(\mathbf{B}_0^{**} + \mathbf{B}_1^{**}\right)\mathbf{x} = \mathbf{Q}\mathbf{x} \tag{2.14}$$

In describing the theoretical long-run multipliers of the model, we have to bear in mind the following points

- We estimate a reduced form model and examine their compatibility with a reasonable and coherent set of assumptions on the likely values of the structural parameters. We do not aim at conducting inference on the deep parameters themselves.
- The theoretical multipliers give indications on the long run behaviour of the system in the case of changes in the exogenous variables. Obviously, it may be the case that the estimated model is not able to correctly pick up certain response patterns: in such case the estimated responses may be "not significant".
- We do not know exactly how long does it take for the system to settle to a new equilibrium but we know that the roots of the theoretical model are: γ,λ, r_e, r_q, r_n.

Now we discuss the theoretical multipliers with respect to changes in each exogenous variable.

Responses to changes in *k*

- $\partial u^* / \partial k^* = 0$
- $\partial (w - p)^* / \partial k^* = \dfrac{1 - \text{a}}{(1 + ?(1 - \text{a}))} > 0$
- $\partial n^* / \partial k^* = \partial l^* / \partial k^* = \dfrac{?(1 - \text{a})}{?(1 - \text{a}) + 1} > 0$, always positive and less than 1.

Responses to changes in x

- $\partial u^* / \partial x^* = 0$
- $\partial (w - p)^* / \partial x^* = 0$

- $\partial n^* / \partial x^* = \partial l^* / \partial x^* = 0$

Responses to changes in rm (m-p)

- $\partial u^* / \partial rm^* = 0$
- $\partial (w - p)^* / \partial rm^* = 0$
- $\partial n^* / \partial rm^* = \partial l^* / \partial rm^* = 0$

The theoretical long run effect of policy changes are all zero.

Responses to changes in UB

- $\partial u^* / \partial UB^* = \dfrac{?}{(1-?)(1-a)} > 0$
- $\partial (w-p)^* / \partial UB^* = \dfrac{-f(1-a)(1-?)+?(1+?)}{(1-?)(1+?(1-a))}$
- $\partial n^* / \partial UB^* = \dfrac{f(1-a)(1-?)-?(1+?)}{(1-?)(1+?(1-a))(1-a)}$
- $\partial l^* / \partial UB^* = \dfrac{f(1-a)(1-?)+?[?(1-a)-?]}{[1+?(1-a)](1-?)(1-a)}$

Note that if λ is close to 1 (not equal to one, otherwise the long run multipliers do not converge), we can say that $\partial(w\text{-}p)^*/\partial UB^*$ is positive and that $\partial n^*/\partial UB^*$ is likely to be positive. When $\lambda \approx 1$, the sign of $\partial l^*/\partial UB^*$ is positive if $? > \dfrac{?}{1-a}$ (when the wage effect dominates the discouraging effect).

Responses to changes in *LT*

- $\partial u^* / \partial LT^* = \dfrac{\mu}{(1-?)(1-a)} > 0$
- $\partial (w\text{-}p)^* / \partial LT^* = \dfrac{?(1-?)(1-a)+\mu(1+?)}{(1-?)(1+?(1-a))} > 0$
- $\partial n^* / \partial LT^* = -\dfrac{?(1-a)(1-?)+\mu(1+?)}{(1-?)(1+?(1-a))(1-a)}$
- $\partial l^* / \partial LT^* = \dfrac{-?(1-a)(1-?)+\mu[?(1-a)-?]}{[1+?(1-a)](1-?)(1-a)}$

If $\lambda \approx 1$, $\partial l^*/\partial LT^*$ is positive if positive if $? > \dfrac{?}{1-a}$ (when the wage effect dominates the discouraging effect).

3. Sources of structural change

Applying our time varying parameter model to the Italian labour market is motivated by the enormous relevance of the unemployment problem both at an academic level (see [5],[6],[7], [2], [3], [22] and [24]) and also from the policy point of view. Moreover, the case of Italy is one of the most peculiar in Europe because of its very high unemployment level, its geographical and sectoral inequalities and the specific role played in Italy by institutions like the Unions and the "Scala Mobile".

Finally, the behaviour of the Italian economic aggregates and, in particular, labour demand and labour supply are a valid case study in order to assess the effectiveness of econometric techniques for the estimation of time varying models. In fact, both smooth transitions and also structural breaks have affected the economic and institutional system in Italy over the period going from 1970 to 1995. A complete summary of these events is not a goal of this paper (at this purpose see [12]), so that we only report here some examples of particular interest.

- Because of a poor commodities endowment, the effects on wages, prices and productivity of the supply (oil) shocks occurred in the 1970s and in the 1980s have been in Italy larger and more relevant than in most of the other european countries. Again, from the macroeconomic point of view, it is worth to remember the Italian departure from the EMS after the massive devaluation of Lira in the 1992.

- A quite common finding of the empirical literature on the Italian case over the two last decades (see [6]) is the change of the elasticities of the production function with respect to capital and labour. In terms of our model this stylized fact, if true, should imply that the parameter α evolves over time so that also the long run equilibrium levels of unemployment and real wages and their sensitivity to shocks on exogenous variables are not constant over the estimation period we consider.
- Many radical transformations have taken place in the Italian labour market over the period 1970-1995. The growing weight of the elderly component on the total population and the slowly increasing female participation into the labour force probably modified the intensity and the direction of the relationship between labour supply and real after-tax wage, producing some consequence on the parameter ξ of our structural model. Also the links between prices and wages within price setting and wage setting equations probably passed through an evolution process due both to the numerous reforms of the wage indexation schemes and also to the continuous changes of the bargaining rules. Moreover, the role played by the Unions (and the dimension of the structural parameter λ) has deeply changed over the 80s: the most popular indicators of Unions performance seem to reveal a reduction of the unions power and a growth of their degree of centralization.
- Finally, from the statistical point of view, various discontinuities and breaks affect the series used to measure the labour market aggregates: *in primis* one should think of the change in the definition of the labour force imposed to the Italian Statistical Office from the need of harmonizing its system to the European Community one.

4. Estimation issues

As is often the case in econometric applications, also in this case there are different viable routes to model estimation. The choice among these alternative options depends of course on the specific goals of the analysis, on the computational resources and on the quality of data.

A first possibility is that to estimate the structural model under all the constraints that arise from the rational expectations hypothesis, according to which the parameters in (2.12) are complicated non linear functions of the parameters in (2.1-2.6). In this way, it is possible to make inference directly on them, and on the cross elasticities of the variables under the analysis.

Alternatively, one can think at expression (2.12) as the "primitive model", i.e. as a system of linear simultaneous equations which can be estimated by means of traditional (*FIML*, *3SLS*) or Bayesian techniques, provided that identification is achieved.

In this case, the deep structural parameters are not directly subject to inference, but rather one estimates the linear form parameters and some particularly relevant functions of them, such as the interim and long run multipliers of exogenous variables and the structural impulse response functions, or the long run equilibrium solution of the model (see 2.14). We are currently investigating this strategy in an improved version of [1].

On the other hand, if one values particularly the need of aptly modelling transition phenomena, a sensible approach is based on time varying parameter framework. In the light of these considerations, our approach is based on the reduced form of the theoretical model, allowing for parameter time variability.

In particular, we estimate a stripped down version of our theoretical model obtained by considering the following variables:

$$\mathbf{y}_t = \begin{bmatrix} w_t - p_t \\ n_t \\ l_t \end{bmatrix} = \begin{bmatrix} rw_t \\ n_t \\ l_t \end{bmatrix} \tag{4.1}$$

$$\mathbf{x}_t = \begin{bmatrix} k_t \\ x_t \\ m_t - p_t \\ UB_t \\ LT_t \end{bmatrix} = \begin{bmatrix} k_t \\ x_t \\ rm_t \\ UB_t \\ LT_t \end{bmatrix}$$

The estimated model is a unrestricted VAR with deterministic and exogenous variables:

$$?^{\,y}(L)\mathbf{y}_t = ?\,\mathbf{d}_t + ?^{\,x}(L)\mathbf{x}_t + \mathbf{e}_t, \mathbf{e}_t \sim VWN(0, \mathbf{H}^{-1}) \qquad (4.2)$$

$$?^{\,y}(L) = \mathbf{I}_n - \sum_{i=1}^{h_1} ?^{\,y}_i L^i, ?^{\,x}(L) = \sum_{i=0}^{h_2} ?^{\,x}_i L^i$$

which can be thought of just as a simplified reduced form of our structural model.

As it will be evident from the methodological section, we adopt Bayesian estimation techniques in order to obtain the joint posterior distributions (and their moments) for any function of the parameters of the VAR we are interested in. In particular we are interested in conducting inference on the coefficients describing the dynamic responses of endogenous variables (*rw*, *n* and *l*) with respect to observable exogenous factors measured by the series *k*, *x*, *rm*, *UB* and *LT*.

These coefficients are obtained as solution of the following identity:

$$\mathbf{y}_t = \mathbf{d}^*_t + \mathbf{C}(L)\mathbf{x}_t + \mathbf{e}_t \qquad (4.3)$$

$$\mathbf{C}(L) = \sum_{i=0}^{\infty} \mathbf{C}_i L^i \;, ?^{\,x}(L) \equiv ?^{\,y}(L)\mathbf{C}(L)$$

5. Methodological section

Let us consider a time varying parameter (henceforth TVP) VAR model (potentially allowing for exogenous and deterministic regressors), characterised by the following density:

$$p(\mathbf{y}_t|\text{b}_t,\mathbf{H})=N((\mathbf{I}_n \otimes \mathbf{z}_t')\text{b}_t,\mathbf{H}^{-1}) \qquad (5.1)$$

$$\text{b}_t = vec(?_t')$$

In the previous expression b_t is the vector collecting all the first order parameters of the model. The state equation for them is:

$$\tilde{ß}_t = \boldsymbol{A}(\text{x})\tilde{ß}_{t-1} + \text{h}_t \qquad (5.2)$$

$$\tilde{ß}_t = \text{b}_t - \underline{\text{b}}_0,$$

$$\text{h}_t \sim \text{NID}(\mathbf{0},\text{W}(\text{x}))$$

The initialisation of the $\boldsymbol{B}_t$ sequence (i.e. the prior distribution for b_0) is:

$$p(\text{b}_0) \sim N(\underline{\text{b}}_0,\mathbf{Q}_0(\text{x})) \qquad (5.3)$$

As customary in the BVAR literature (see for instance [14]), there is a vector of hyperparameters x controlling the specification of the TVP mechanism and the second moments of the initialisation of the state vector. The vector x is as follows:

$$? = [p_1, p_2, p_3, p_4, p_5, p_6, p_7]' \qquad (5.4)$$

These hyperparameters determine:

$$\boldsymbol{A}(\text{x}) = \pi_7 \mathbf{I}$$

$$\text{W}(\text{x}) = \pi_6 \cdot \boldsymbol{Q}_0$$

and $\boldsymbol{Q}_0$ is a diagonal matrix whose diagonal elements are determined as follows

$$[\text{var(coeff.on r}^{\text{th}}\text{ lag of i}^{\text{th}}\text{ end. var. in i}^{\text{th}}\text{ eq.)}]^{1/2} = p_1 \cdot p_5^{-r}$$

$$[\text{var(coeff.on r}^{\text{th}}\text{ lag of j}^{\text{th}}\text{ end. var. in i}^{\text{th}}\text{ eq.)}]^{1/2} = p_1 \cdot p_2 \cdot p_5^{-r}$$

$$[\text{var(coeff.on r}^{\text{th}}\text{ lag of j}^{\text{th}}\text{ exog. var. in i}^{\text{th}}\text{ eq.)}]^{1/2} = p_1 \cdot p_3 \cdot p_5^{-r}$$

$$[\text{var(coeff.on r}^{\text{th}}\text{ lag of j}^{\text{th}}\text{ det. var. in i}^{\text{th}}\text{ eq.)}]^{1/2} = p_1 \cdot p_4$$

The initial state mean is specified as in the customary BVAR literature: the coefficients on first lag of dependent variable in each equation are given unit prior mean, whereas all other coefficients are assigned zero prior mean.

We assign a prior distribution to x as follows:

$p(\text{x})\sim NT(\underline{\text{x}},\underline{\mathbf{H}}_{\text{X}}^{-1})$ (5.5)

where *NT* denotes a normal distribution truncated to the support X.

The prior distribution for $\mathbf{H}=\text{S}^{-1}$ is Wishart**:**

$p(\mathbf{H})\sim W(\underline{\text{v}},\underline{\mathbf{S}})$ (5.6)

The strategy we propose in order to conduct inference is to use Monte Carlo integration of the model by using MCMC. A feasible approach is based on sequentially drawing the conditional posterior distributions of proper subsets of (hyper)parameters (Gibbs sampling), as follows.

A) The conditional distribution of $\underline{\text{b}}_T=[\text{b}_0',\text{b}_1',...\text{b}_T']'$ can be easily obtained by means of the Kalman filter, and using a recursive partition:

$$p(\underline{\mathbf{ß}}_t \mid \mathbf{y},\mathbf{H},\mathbf{?}) = p(\mathbf{ß}_t \mid \mathbf{y},\mathbf{H},\mathbf{?}) \cdot \prod_{t=0}^{T-1} p(\mathbf{ß}_t \mid \underline{\mathbf{ß}}^{t+1},\mathbf{y},\mathbf{H},\mathbf{?}) \qquad (5.7)$$

$$p(\mathbf{ß}_T \mid \mathbf{y},\mathbf{H},\mathbf{?}) \sim N(\overline{\mathbf{ß}}_{T|T},\overline{\mathbf{H}}_{b_{T|T}}^{-1}) \qquad (5.8)$$

$$\underline{\mathbf{ß}}^{t+1} = [\mathbf{ß}_{t+1}',\mathbf{ß}_{t+2}',...,\mathbf{ß}_T']' \qquad (5.9)$$

B) Random draws may be easily obtained from the conditional distribution of **H** which has a Wishart form**:**

$$p(\mathbf{H} \mid \mathbf{y},\underline{\mathbf{ß}}_T,\mathbf{?}) = W(\overline{n},\overline{\mathbf{S}}) \qquad (5.10)$$

where the moments $\overline{n},\overline{\mathbf{S}}$ are calculated on the basis of the usual results of the conditionally conjugated analysis, as follows. Following the notation in ([4], p.139), a Wishart distribution for ***H***, an $(n\times n)$ symmetric and positive definite random matrix, is:

$$p(\mathbf{H}\mid \underline{n},\underline{\mathbf{S}}) = W(\underline{n},\underline{\mathbf{S}}) = c\cdot \mid \mathbf{H}\mid^{[\underline{n}-(n+1)]/2}\cdot\exp\left[-tr(\mathbf{H}\underline{\mathbf{S}})\right]$$

$$c=\frac{\mid \underline{\mathbf{S}}\mid^{\underline{n}}}{\Gamma_n(\underline{n})},\Gamma_n(\underline{n})=p^{\frac{n(n-1)}{4}}\prod_{i=1}^{n}\Gamma\left(\frac{2\underline{n}+1-i}{2}\right)$$

$$E(\mathbf{H})=\underline{n}\cdot\underline{\mathbf{S}}^{-1},E(\mathbf{H}^{-1})=\left(\underline{n}-\frac{n+1}{2}\right)^{-1}\underline{\mathbf{S}}$$

The data density, given $\underline{\text{b}}_t$, the full path of state vectors, and x, the full hyperparameters set, can be seen as the following function of **H**:

$$p(\underline{\mathbf{y}}_t \mid \underline{\mathbf{ß}}_t,\mathbf{?}) \propto \mid \mathbf{H}\mid^{T/2}\exp\left\{-\frac{1}{2}tr(\mathbf{HS})\right\}$$

$$\mathbf{S}=\sum_{t=1}^{T}\mathbf{e}_t\mathbf{e}_t',=\mathbf{y}_t-(\mathbf{I}_n\otimes\mathbf{z}_t')\mathbf{ß}_t$$

By combining the data density and the prior, it is immediately obtained:

$$p(\mathbf{H}\mid \underline{\mathbf{y}}_t,\underline{\mathbf{ß}}_t,\mathbf{?}) = W_n(\mathbf{H}\mid \overline{n},\overline{\mathbf{S}}),$$

$$\overline{n}=\underline{n}+T,\overline{\mathbf{S}}=\mathbf{S}+\underline{\mathbf{S}}$$

C) The conditional posterior distribution of x has not a standard form, but it may be simulated through Metropolis-Hastings, given a proper choice of the candidate function.

As for the forecasts, it is straightforward to estimate, at the desired level of precision, whatever synthesis measure (conditional or not) of the posterior density of the forecasts, which takes into proper account all the uncertainty sources.

5.1 Kalman filter

Let us suppose to consider a time varying parameter VAR model (potentially allowing for exogenous and deterministic regressors), characterized by the following density:

$p(\mathbf{y}_t|\text{b}_t,\mathbf{H})=N((\mathbf{I}_n\otimes\mathbf{z}_t')\text{b}_t,\mathbf{H}^{-1})$ (5.11)

$\text{b}_t=vec(\mathbf{B}_t')$

This is the measurement equation. In the previous expression b_t is the vector collecting all the first order parameters of the model. The state equation for them is:

$b_t=\mathbf{A}(x)b_{t-1}+h_t$

$\tilde{\mathbf{ß}}_t = b_t-\underline{b}_0,$

$h_t \sim NID(\mathbf{0},\underline{\mathbf{K}}^{-1}(x))$

The initialisation of the $\boldsymbol{B}_t$ sequence (prior distribution for b_0) is:

$p(b_0)=N(\underline{b}_0,\underline{\mathbf{H}}^{-1}(x))$

The combination of prior and sample information on a parameter vector may be obtained also by performing the Kalman filter (see [17], chapter 13): the aim is to update the inference on an unobservable state variable whenever new information on the observed measurement variables become available. The equations above define the state-space representation of the model. The *measurement equation* (5.11) links the observed variable ($\mathbf{y}_t$) to the unobserved state vector b_t in a linear way through a set of explanatory variables. Equation (5.12), the *state equation*, shows how the state vector evolves over time; equation (5.13) provides a guess formulated at *t-1* on the state of the system at the same instant. Referring this equation at time *t*=1 gives the initialisation of the filter. The Kalman filter operates via the cyclical repetition of the *extrapolation* step:

$b_{t|t-1} = \mathbf{A}b_{t-1|t-1}+(\mathbf{I}_k-\mathbf{A})\underline{b}_0,$

$\mathbf{Q}_{t|t-1} = \mathbf{A}\mathbf{Q}_{t-1|t-1}\mathbf{A}'+W$

$\mathbf{Q}_{t|t-1}^{-1/2}b_t = \mathbf{Q}_{t|t-1}^{-1/2}b_{t|t-1}+\mathbf{Q}_{t|t-1}^{-1/2}\mathbf{e}_{t|t-1}$

and the *updating* step, on the basis of the available information on y_t e $\mathbf{z}_t$ via the measurement equation:

$\mathbf{H}^{1/2}\mathbf{y}_t=\mathbf{H}^{1/2}(\mathbf{I}_n \otimes \mathbf{z}_t')b_t + \mathbf{H}^{1/2}e_t$

These two expressions can be combined to obtain[10]:

$p(b_t|\underline{\mathbf{y}}_t,g,x) = N(b_{t|t},\mathbf{Q}_{t|t})$

$\mathbf{Q}_{t|t} = [\mathbf{Q}_{t|t-1}^{-1}+(\mathbf{I}_n\otimes\mathbf{z}_t)\mathbf{H}(\mathbf{I}_n\otimes\mathbf{z}_t')]^{-1}$

$b_{t|t} = \mathbf{Q}_{t|t}[\mathbf{Q}_{t|t-1}^{-1}b_{t|t-1}+(\mathbf{I}_n \otimes \mathbf{z}_t)\mathbf{H}\mathbf{y}_t]$

5.2 Likelihood function

As a by-product of the KF, it is possible to obtain the likelihood of each observation $p(\mathbf{y}_t|\mathbf{y}^*_{t-1},H,x)$:

$p(\mathbf{y}_t|\mathbf{y}^*_{t-1},\mathbf{H},x) = N(\boldsymbol{\mu}^y_{t|t-1},\mathbf{S}^y_{t|t-1})$

$\boldsymbol{\mu}^y_{t|t-1} = (\mathbf{I}_n\otimes \mathbf{z}_t')b_{t|t-1},$

$\mathbf{S}^y_{t|t-1} = (\mathbf{I}_n\otimes\mathbf{z}_t')\mathbf{Q}_{t|t-1}(\mathbf{I}_n\otimes\mathbf{z}_t)+\mathbf{H}^{-1}$

$\mathbf{y}^*_{t-1} = \{y_t, \mathbf{z}_k: t = 1-p, ..., t-1, k = 1-p, ..., t\}$

($\hat{\mathbf{ß}}_{t|t-1}$ and $\boldsymbol{Q}_{t|t-1}$ are moments of b_t conditional on $\underline{\boldsymbol{y}}_{t-1}^*$,$\boldsymbol{z}_t$ is not stochastic if we condition on $\underline{\boldsymbol{y}}_{t-1}^*$ and e_t is Gaussian with constant conditional and unconditional moments).

In other words:

$$\ln\ p(\mathbf{y}_t\ |\underline{\mathbf{y}}^*_{t-1},\mathbf{H},?) = -\frac{n}{2}\ln(2p) - \frac{1}{2}\ln|\mathbf{S}^y_{t|t-1}| - \frac{1}{2}(\mathbf{y}_t-\boldsymbol{\mu}^y_{t|t-1})'\left[\mathbf{S}^y_{t|t-1}\right]^{-1}(\mathbf{y}_t-\boldsymbol{\mu}^y_{t|t-1})$$

Note that in expression above, the second and third addenda depend on **H** and x.

Taking all this into consideration, one obtains the log-likelihood function of the model as:

$$\ln\ p(\underline{\mathbf{y}}_t\ |\mathbf{H},?) = -\frac{nT}{2}\ln(2p) - \frac{1}{2}\sum_{t=1}^{T}[\ln|\mathbf{S}^y_{t|t-1}| + (\mathbf{y}_t-\boldsymbol{\mu}^y_{t|t-1})'\left[\mathbf{S}^y_{t|t-1}\right]^{-1}(\mathbf{y}_t-\boldsymbol{\mu}^y_{t|t-1})$$

5.3 Multi-move simulation of the state vector

Following ([10]), we can factorise the full conditional posterior distribution of the state vectors belonging to the sample period as in expression (5.7); the typical term in that expression is:

$$p(\mathbf{ß}_t \mid \underline{\mathbf{ß}}^{t+1}, \underline{\mathbf{y}}_T, \mathbf{H}, \mathbf{?}) = \frac{p(\mathbf{ß}_t \mid \underline{\mathbf{y}}_t, \mathbf{H}, \mathbf{?}) \cdot p(\underline{\mathbf{y}}^{t+1}, \underline{\mathbf{ß}}^{t+1} \mid \underline{\mathbf{y}}_T, \mathbf{ß}_t \mathbf{H}, \mathbf{?})}{\int p(\mathbf{ß}_t \mid \underline{\mathbf{y}}_t, \mathbf{H}, \mathbf{?}) \cdot p(\underline{\mathbf{y}}^{t+1}, \underline{\mathbf{ß}}^{t+1} \mid \underline{\mathbf{y}}_T, \mathbf{ß}_t \mathbf{H}, \mathbf{?}) d\mathbf{ß}_t}$$

The second term in the numerator of the expression above can be further decomposed into:

$p(\underline{\mathbf{y}}^{t+1}, \underline{\mathrm{b}}^{t+1} | \underline{\mathbf{y}}_t, \mathrm{b}_t, \mathbf{H}, \mathrm{x}) = p(\mathrm{b}_{t+1} | \mathrm{b}_t, \underline{\mathbf{y}}_t, \mathrm{x}) \cdot p(\underline{\mathbf{y}}^{t+1}, \underline{\mathrm{b}}^{t+2} | \underline{\mathbf{y}}_t, \mathrm{b}_t, \mathrm{b}_{t+1}, \mathbf{H}, \mathrm{x}) =$

$= p(\mathrm{b}_{t+1} | \mathrm{b}_t, \underline{\mathbf{y}}_t,) \cdot p(\underline{\mathbf{y}}^{t+1}, \underline{\mathrm{b}}^{t+2} | \underline{\mathbf{y}}_t, \mathrm{b}_{t+1}, \mathbf{H}, \mathrm{x})$

where the second factor does not depend on b_t, given Markovian property of the state vector. Therefore we have:

$$p(\mathbf{ß}_t \mid \underline{\mathbf{ß}}^{t+1}, \underline{\mathbf{y}}_T, \mathbf{H}, \mathbf{?}) = \frac{p(\mathbf{ß}_t \mid \underline{\mathbf{y}}_t, \mathbf{H}, \mathbf{?}) \cdot p(\mathbf{ß}_{t+1} \mid \mathbf{ß}_t, \underline{\mathbf{y}}_t, \mathbf{H}, \mathbf{?})}{\int p(\mathbf{ß}_t \mid \underline{\mathbf{y}}_t, \mathbf{H}, \mathbf{?}) \cdot p(\mathbf{ß}_{t+1} \mid \mathbf{ß}_t, \underline{\mathbf{y}}_t, \mathbf{H}, \mathbf{?}) d\mathbf{ß}_t} =$$

$$= \frac{p(\mathbf{ß}_t, \mathbf{ß}_{t+1} \mid \underline{\mathbf{y}}_t, \mathbf{H}, \mathbf{?})}{\int p(\mathbf{ß}_t, \mathbf{ß}_{t+1} \mid \underline{\mathbf{y}}_t, \mathbf{H}, \mathbf{?}) d\mathbf{ß}_t}$$

From the Kalman Filter we have:

$$p\left[\begin{pmatrix} \mathbf{ß}_t \\ \mathbf{ß}_{t+1} \end{pmatrix} \underline{\mathbf{y}}_t, \mathbf{H}, \mathbf{?} \rangle\right] = N\left[\begin{pmatrix} \hat{\mathbf{ß}}_{t|t} \\ \hat{\mathbf{ß}}_{t+1|t} \end{pmatrix}, \begin{pmatrix} \mathbf{Q}_{t|t} & \mathbf{Q}_{t|t} \cdot \mathbf{A}' \\ \mathbf{A} \cdot \mathbf{Q}_{t|t} & \mathbf{Q}_{t+1|t} \end{pmatrix}\right]$$

Due to the Gaussianity of all error terms, we can readily obtain the desired distribution:

$p(\mathrm{b}_t | \underline{\mathrm{b}}^{t+1}, \underline{\mathbf{y}}_T, \mathbf{H}, \mathrm{x}) = N(\hat{\mathbf{ß}}_{t|t+1}, \mathbf{Q}_{t|t+1})$

$$\hat{\mathbf{ß}}_{t|t+1} = \hat{\mathbf{ß}}_{t|t} + \mathbf{Q}_{t|t} \mathbf{A}' \mathbf{Q}_{t+1|t}^{-1} \left(\mathbf{ß}_{t+1} - \mathbf{ß}_{t+1|t} \right)$$

$$\mathbf{Q}_{t|t+1} = \mathbf{Q}_{t|t+1} - \mathbf{Q}_{t|t+1} \mathbf{A}' \mathbf{Q}_{t+1|t}^{-1} \mathbf{A} \mathbf{Q}_{t|t}$$

So these conditional distributions can be sequentially simulated in order to obtain a draw from the distribution:

$p(\underline{\mathrm{b}}_T | \underline{\mathbf{y}}_T, \mathbf{H}, \mathrm{x})$

basically using the results of the application of the Kalman Filter.

6. Results from the estimated model

In this section we report the results of the estimation and simulation of our model, as specified in section (4). The model has real wage (*rw*), employment level (*n*) and participation (*l*) as endogenous variables. The exogenous variables set includes the same five components as the theoretical model, but the money stock is expressed in real terms The source of the data is described in the Appendix.

It is worth stressing that the adopted specification allows for 2 lags of the endogenous variables; as for the exogenous regressors, only their current values are included. As for the deterministic variable, we only included an intercept term.

Figures (6.1.1) to (6.5.4) contain the cumulated response functions (henceforth RFs) of *rw*, *n* and *l* with respect to shocks on the exogenous variables. We decided to present only the cumulated RFs, because we aim at analysing the effects of long lasting modifications in the exogenous variables.

Moreover, since our goal is to find out the determinants of unemployment hysteresis, in commenting the results we will look mainly at the long run behaviour of the labour indicators, having always in mind the suggestions coming from the theoretical dynamic multipliers.

6.1 Capital changes

In general, the long run effects of a unit permanent increase of capital stock are largely consistent with the indications coming from the theoretical model: the unemployment level does not seem to be affected by this change at any horizon, whereas the behaviour of the real wages is characterised by persistent growth, which should be the counterpart of the increased productivity.

More in detail, the growth of capital triggers an increase of the real wage and a global expansion of economic activity, pushing permanently up employment . The encouragement process generated by this effect, jointly with the increase in labour income, is the determinant of the increase in participation.

It is worth to note that the employment increase is a bit larger than the one in *l* throughout the whole simulation horizon, so that unemployment is slightly declining, but not in a significant way.

As a matter of international comparison, if we compare the results for the Italian model with the ones obtained for the US (see [1]), similar to the Italian one for employment, but non-significance of the real wage reaction to the shock.

We believe that the diversity of responses between the Italian and the US case is generated by the different role played by the unions in the two countries. A synthetic measure of this is the ratio between the union coverage index and the degree of coordination in the wage bargaining activity. Over the considered period (1975-1995), Italy is characterised by high values of this ratio (see [1]): in these circumstances, unions activity is particularly intrusive and they are able to turn the increased productivity into a real wage growth; this is a typical result of the insider/outsider model.

On the contrary, when the bargaining activity is more coordinated, or the coverage index is very low, like in the USA, unions are not able to oppose firms determination to employ higher quantity of labour, but keeping wages fixed. The final consequence is an increase in profits for the firms, getting the whole benefit generated by the productivity growth.

6.2 Aggregate demand modifications

At a theoretical level, demand shocks are not expected to produce real effects on the labour market in the long run. This is exactly the empirical evidence for USA (see [1]), where the RFs of real wage, employment and participation do not show any persistent long run reaction with respect to fiscal shocks. The same is true also in the short run. This could provide some evidence in favour of a very small value of the structural parameter σ.

A strong hysteresis phenomenon seems to drive the behaviour of the Italian labour market (see figures 6.2.1 to 6.2.4): the employment evolution is positively affected both in the short and in the long run whenever either the fiscal and the monetary stance of the Italian policies become looser. The long run effects of the two shocks are more or less equivalent in terms of their orders of magnitude, but the profile of the RF to the former one is at first characterized by an overshooting behaviour.

Some differences arise for the real wage and the labour force: the former exhibits a relevant long run growth only in consequence of a monetary perturbation due also to the fact that a weakly positive labour force response is not able to overcome the labour demand excess. This picture is overturned in the case of a fiscal expansion: the profile of the labour force reaction is very similar to the employment one, but a little bit lagged and slightly smaller, giving rise to a weak and not significant unemployment reduction.

We see a substantial irresponsiveness of the real wage, confirming that inflows and outflows from the labour force are essentially triggered by past and current employment values and not by the expected labour yield.

The rationale of these results is as follows: since ([8]), it became common knowledge that hysteresis phenomena are stronger in economies characterised both by a strongly intrusive role of the Unions and by a high percentage of the long run unemployed on total unemployment. Such features are typical of the European countries, and among them Italy, for which we find a strong evidence of hysteresis. On the contrary, in those economies, like Sweden and USA, in which the unions are powerless or strongly coordinated and the long run unemployment phenomenon is negligible, a very low probability is assigned to the hysteresis event (see [1]).

6.3 Changes in institution variables

6.3.1 Unemployment Benefits

In the Italian case (figures 6.4.1 to 6.4.4), a unit growth of *UB* generates a strongly negative and permanent effect on *n*. This result is compatible with λ close to one, and with positive and high value of γ, being a further sign of the existence of an hysteresis mechanism.

Another potential rationale for a negative relationship between *n* and *UB*, at a given and fixed wage level, is that *UB* is a powerful tool for politicians; whenever unemployment increases, a practical way to obtain political consensus of the newly unemployed workers is to push up their benefits. In this sense, the *UB* variable can be interpreted as being not fully exogenous.

Differently from the suggestions of the theoretical model, no significant reactions are produced on the real wage, probably because the employment reduction prevents the surge of reservation wage from generating a corresponding surge in the effective market wage. Finally, it is worth to notice that there is no response at all by *l*: a possible explanation refers to the fact that the outflows should be offset by the incentive to "stay in" in order to reap higher benefits, whereas, at the same time, a lower *n* should trigger discouragement and curb labour force inflows.

To sketch again some international comparisons (see [1]), in the USA an increase of the replacement ratio has a very strong and persistently positive effect on wages, as suggested by the model. In fact, given the high level of jobs turn-over, there is a strong probability for a temporarily outsider worker to become an insider one, which raises the market power of the non unionised workers and makes the growth of the replacement ratio effective on the true real wage.

6.3.2 Labour Taxes

Our evidence is only partially in line with the findings coming from other recent empirical studies (see[13]), supporting the view that labour taxes reduce labour demand (creating unemployment) and this occurs in a more pronounced way when and where unions are both powerful and decentralised.

This view is essentially confirmed by our estimated model. As for Italy, our evidence confirms that a growing fiscal pressure on labour income persistently pushes up the unemployment rate, given that it is commonly held that decentralised Unions play a relevant role.

Again, it is worth stressing the heterogeneity of the determinants of the unemployment changes when they occur, because this could have different implications in terms of policy intervention. In Italy a growth of *LT* does not reduce significantly employment, but it triggers an increase of participation, which is the main source of the unemployment expansion. The labour force increase is probably a consequence of a high value of λ, and of the fact that $\xi > \dfrac{\gamma}{1-\alpha}$.

What is really hard to interpret is our finding that the increase in taxation seems to induce a significant and long lasting reduction in real wages. In the theoretical model we use as reference there is no way to account for this phenomenon in a formal way. Informally (and completely outside the model), we can interpret this fact as due to a low degree of competition in the goods market, allowing firms to shift the tax burden on consumers by increasing prices.

Jointly considered, the patterns of the responses of *n*, *l* and *rw* for Italy are self-consistent only if three "structural" implications do hold:

1) Positive value of the parameter ξ, reflecting that a higher expected wage level and a lower level of taxation are two incentives for participation;

2) highly positive value for λ (close to 1), reflecting the existence of an hysteresis phenomenon.

Quoting again some international comparisons, in USA, where the degree of the goods market competition is higher than in Italy (Table 3.1 in [16]), the wage reduction is weak and remains not significant throughout the whole simulation horizon. In this way, firms have to reduce employment in a permanent way, in order to counterbalance the growth of the labour costs and to avoid profit reduction. The labour force reaction mimics the employment one, being negative, but it is weakly significant only in the short run.

7. A new proposal for treating structural change

7.1 Methodology

In the traditional BVAR setting, the elements of χ, the hyperparameters vector, are usually calibrated on the basis of the model's forecasting properties, and a (rough) Bayesian interpretation of this procedure is to assume that the researcher has a diffuse prior for the hyperparameters.

A subset of this hyperparameters is particularly important for the econometric treatment of transition/structural change phenomena. These hyperparameters, which we indicate with χ_1, determine W_i, the variance covariance matrix of the transition equation error terms for each equation of the VAR. *Coeteris paribus*, it is evident that, if we consider two possible configurations for W_i, say W_{i0} and W_{i1}, with W_{i1}-W_{i0} positive definite, by using W_{i1} we have a potential higher time variability of the parameters, than that produced by W_{i0}.

In general, in the BVAR approach, hyperparameters are calibrated and costant for all the sampling period. We believe that, in order to successfully model gradual transition phenomena, it is necessary to use a specification in which hyperparameters define a time varying W_i matrix:

$$W_{it}=W_i(\chi_1,t) \qquad (7.1)$$

As a simple example, let us assume that the model at hand has only one parameter, α_t, with a transition equation affected by the error term η_t. The variance of η_t is ω_t, and it is determined by the usual hyperparameters vector, as described in section (5), but with the following modification:

$$w_t = [w_0 + q_1 \cdot s_t \cdot (t-T_0)^{q_2} \cdot \exp(-q_3 \cdot (t-T_0))] \cdot q_0, \qquad (7.2)$$

$$s_t = \begin{cases} 0 \\ 1 \end{cases}, q_1 > 0, q_2 > 0, q_3 > 0, T_0 = \text{last period in which } s_{T_0} = 0.$$

We call this kind of model DVI (Dynamic Variability Intensity). Note that in this way we have a discrete state variable, s_t, which we can consider as an indicator variable associated to the state of low (s_t=0) or high (s_t=1) parameters variability. The hyperparameters in the vector q = $[q_1 q_2\ q_3]'$ define in which way the potential variability of the parameters is allowed to increase, via a corresponding increase in the transition equation error terms variance, and in which way this variance evolves through time (see figure 7.1, showing some possible degrees of time evolution of ω_t).

In our view, some aspects deserve special attention. Firstly, it is necessary to establish in which state (s_t=0 or s_t=1) the system is in each sample (or post-sample) observation. This could in principle be achieved in two different ways:

a) it is possible to impose that the system moves from one state to the other in connection to specific events, such as, for example, the transition from a higher to a lower wage indexation scheme. This entails imposing dogmatic priors on the state of the system for the different observations.

b) It is possible to treat s_t as a unobservable variable, with some transition properties (i.e Markovian), and let the model itself decide how to assign each observation to different states, via application of an apt filter (see [17] and [23]), according to the smoothed probabilities.

Another problem is that of determining the hyperparameters. We believe that the best way is to treat the model in hierarchical terms, as in [11], and to verify whether this approach yields good properties for the estimated models.

7.2 Choice among competing models

This new proposal of tuning the parameters time variability intensity has to be compared with the traditional (uniform degree of time variability)methodology on the grounds of the relative applied properties. In Bayesian terms, the choice among two (or more) non necessarily nested competing models, M_1 and M_2, is accomplished by constructing the posterior odds ratio (*POR*) of the models:

$$POR_{1:2} = \frac{p(M_1 \mid \mathbf{y})}{p(M_2 \mid \mathbf{y})} = \frac{p(M_1)}{p(M_2)} \cdot BF_{1:2} \qquad (7.3)$$

$$BF_{1:2} = \frac{p(\mathbf{y} \mid M_1)}{p(\mathbf{y} \mid M_2)}$$

$$p(\mathbf{y} \mid M_i) = \int_{?_i} p(\mathbf{?}_i) \cdot p(\mathbf{y} \mid \mathbf{?}_i, M_i) d\mathbf{?}_i$$

The magnitude $p(\mathbf{y}|M_i)$ is called *marginalised likelihood* (henceforth ML) of model M_i,i=1,2. In this case, we compare two models:

1. M_1=BVAR model with DVI
2. M_2=standard BVAR model

Note that M_2 is nested within M_1, just by imposing that the hyperparameters vector q (controlling DVI) is equal to a vector of zeros. If we assign to each model equal prior probabilities, the *POR* coincides with the Bayes factor $BF_{1|2}$. Hence, in order to compare different models, it is necessary to compute the relevant MLs. The evaluation of MLs is generally very hard in most applications, as documented in ([15]). For this reason, we resort to the following asymptotic approximation (see [4], p.487):

$$-2\ln(BF_{1:2}) = k_{1:2} - d_{1:2} \qquad (7.4)$$

$$k_{1:2} = -2 \cdot \ln\left(\frac{p(\mathbf{y} \mid M_1, \mathbf{?}_1^*)}{p(\mathbf{y} \mid M_2, \mathbf{?}_2^*)} \right)$$

$$\mathbf{?}_i^* = \arg\max\left\{ p(\mathbf{y} \mid M_i, \mathbf{?}_i) \right\}, i = 1{,}2.$$

where λ_i (i=1,2) is the vector of all parameters and hyperparameters of model M_i and $d_{1:2}$ is the difference of dimensions between l_1 and l_2 (in our case, $d_{1:2}$=3).

Hence, using the criterion (7.4) entails to evaluate Schwartz's BIC criterion for both models:

BIC_i=$-2\cdot\ln(p(y|M_i,\lambda_i^*))+l_i\cdot\ln(T)$

and to choose the model that minimises the BIC. The finite sample properties of approximation (7.4) in this context are unknown; in future research, we intend to use exact simulation techniques in order to directly evaluate the MLs.

7.3 An application to the labour market model

In this subsection we briefly show the results of an application of the *DVI* methodology to the labour market model defined by equations (2.1-2.6). The *prior* depends on a very small set of hyperparameters collected in the vector x as follows:

$$x = \begin{bmatrix} p \\ q \end{bmatrix} = [p_1, p_2, p_3, p_4, p_5, p_6, p_7, q_1, q_2, q_3] \qquad (7.6)$$

We define two different versions of a BVAR on the labour market : the first one is a *DVI* version, whereas the constraint q_1=0 (and q_2=q_3=0) is imposed to the second one, so that the existence of a *DVI* mechanism is excluded. In both cases the optimal hyperparameters configuration is obtained by maximizing the loglikelihoods of each model. In the case of the *DVI* model we *a-priori* suppose that the starting point of the period characterized by greater turbulence is located in 1990Q1 while the maximum variability intensity point is 1992Q4 [11]. These hypothesis are summarized by the following initialisation values: q_1=0.0000001, q_2=1.2, q_3=0.1.

The comparison of the two models, based on the value of the approximate log-Bayes Factor reported in table 7.1, clearly suggests a better performance for the *DVI* model.

Table 7.1: approximate log-Bayes Factor

DVI model *vs*. No DVI model	-8.10

The profile of the estimated time evolution of parameters variability is picted in figure 7.2: the maximum intensity point is just the one supposed a priori. The period of larger turbulence seems to die out by the end of 1995.

8. Conclusions

In this paper, we use a dynamic multivariate time varying parameter model to investigate the determinants of unemployment persistence.

From the methodological viewpoint, we modify the existing BVAR methodology in two directions. First of all, we treat our model, giving a weakly informative prior on the hyperparameters of the model. The resulting posterior distribution is analysed by means of an aptly defined MCMC sampling strategy.

Secondly, we present some preliminary evidence on a new proposal to deal with transition phenomena, which is based on a modified specification for the degree of time variability of the parameters. The preliminary results from a pilot application in this regard suggest that it might be profitable to use such modified approach on our model, and that the turbulence phase is over by the end of the sample period.

The application to the labour market model with a hierarchical structure delivers some notable results. From the economic viewpoint, our empirical evidence is in accordance with the theory in showing that also shocks to institution variables may influence unemployment in the long run. Nevertheless, considering the role of institutions requests a large set of further clarifications. As for the role played by unions, our evidence suggests that this can be synthetically measured by the ratio between the contracts coverage index and the degree of bargaining coordination. In any case, even when this ratio is quite high, unions seem to be able (in the long run) to defend wages and the employed *status* of their members only when this does not reduce the firms profits. This is the case when there is a growth of the capital stock or, more generally, an increase in labour productivity, but not when fiscal pressure increases. In this case, the higher tax level is accompanied to a reduction of real wages, and this is hard to reconcile with the theoretical model.

In other countries, such as USA, the goods markets are characterised by a higher degree of competition which prevents significant price-making power by firms, so that the final effect is a reduction of employment .

We do not intend to propose any miraculous remedy to any unemployment persistence in addition to those contained in the large literature on this topic. This would be a useless operation. Nevertheless, on the basis of our empirical findings, a simple and general, but often ignored, methodological rule may be suggested to deal with the problem. The crucial points are the interaction between shocks and institutions, already stressed in [6] and [9], and, overall, the interactions among the different institutions. The effects on unemployment of changing one of them depend on the size and the features of all the others. Our results seem to cautiously suggest that, for example, a reduction of labour taxes will have different consequences on the unemployment level according to the intrusiveness of unions, to the degree of competitiveness in the goods markets, to the wage elasticity of labour demand and labour supply functions, and to the unemployment benefits regime.

As a consequence, no partial policy measure, acting on a single aspect of the question, may be effective in solving the problem. Only a general approach, producing a thorough and full reform of labour markets, goods markets and welfare state is bound to get the results it aims for.

Anyway, this paper is only the starting attempt to investigate the usefulness of time varying parameter models on the labour market and the different future lines of research are open. From the practical point of view, it would be useful to repeat the analysis in per capita terms, after normalizing all the involved variables by a population measure; moreover, an evaluation of the robustness of the results with respect to different measures of fiscal and monetary policy is needed.

From the methodological point of view, many things need to be investigated. First of all, we still have to provide a careful sensitivity analysis of our results with respect to the prior. Secondly, we do not have performed a fully MCMC based approach on the DVI model and the model selection procedure used in the paper is still based on an approximation. Exact methods will be developed in the near future.

Figure 6.1.1:response of wr to k

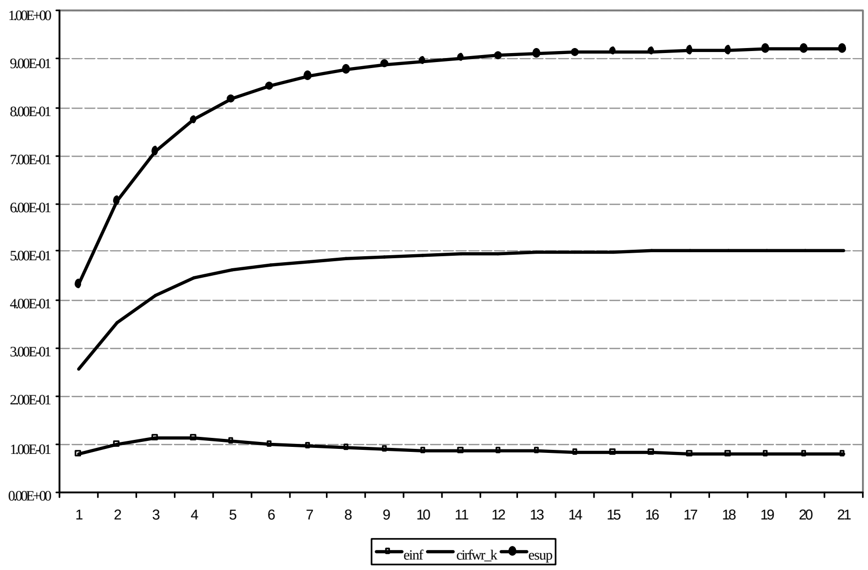


Figure 6.1.2:response of n to k

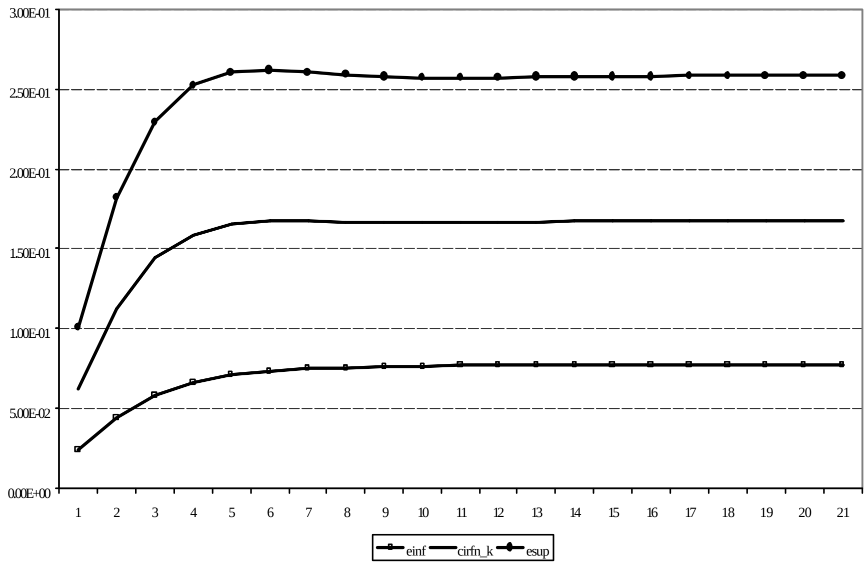


Figure 6.1.3:response of l to k

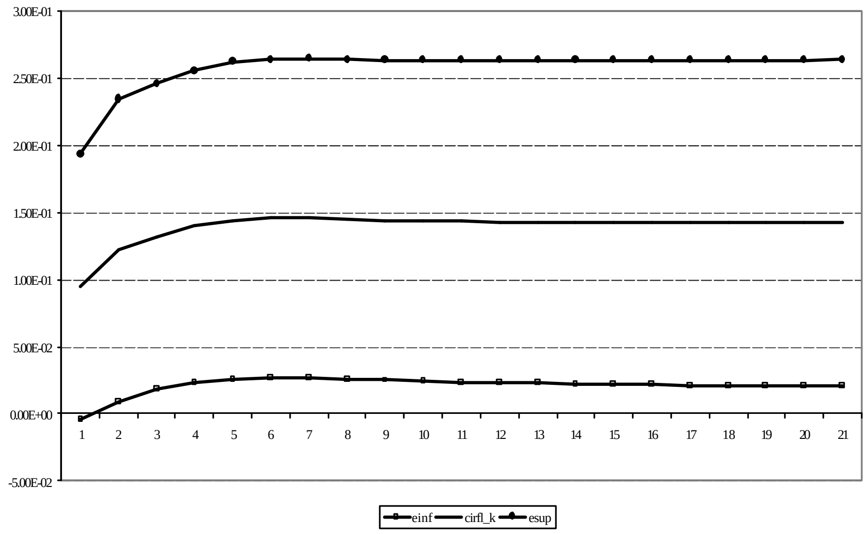


Figure 6.1.4:response of u to k

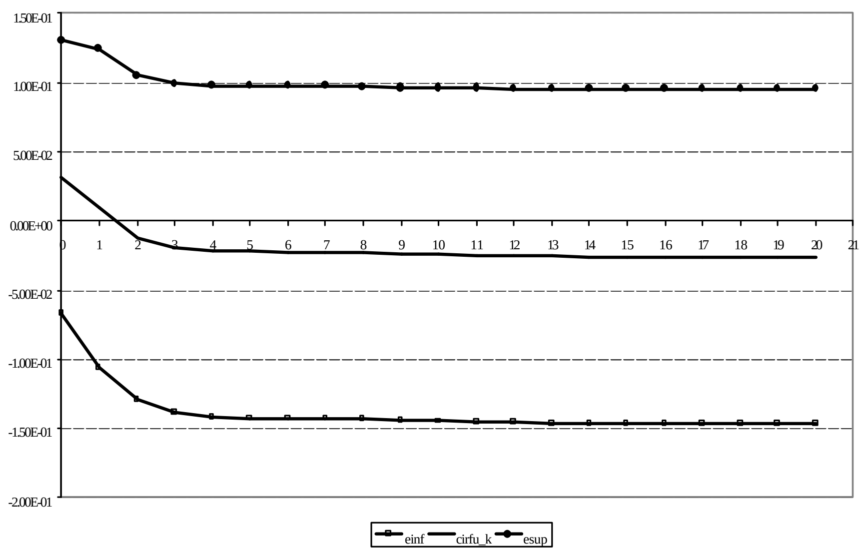


Figure 6.2.1:response of wr to x

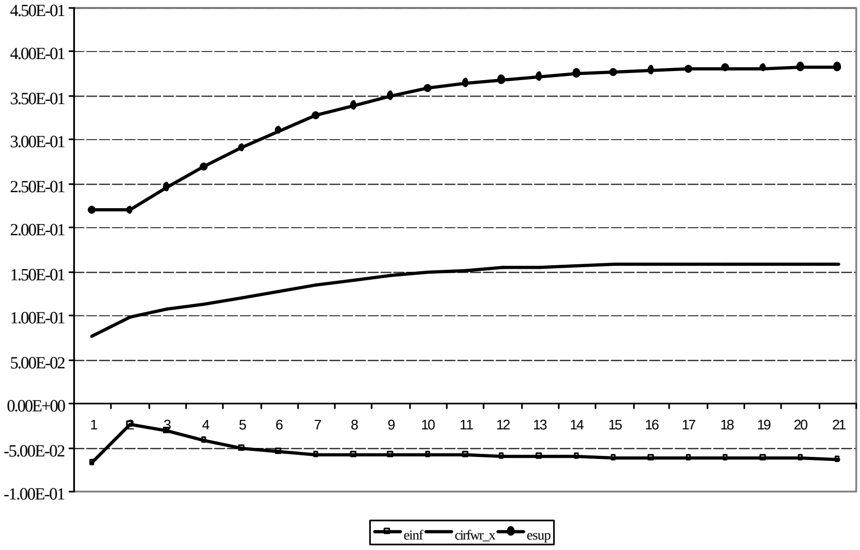


Figure 6.2.2:response of n to x

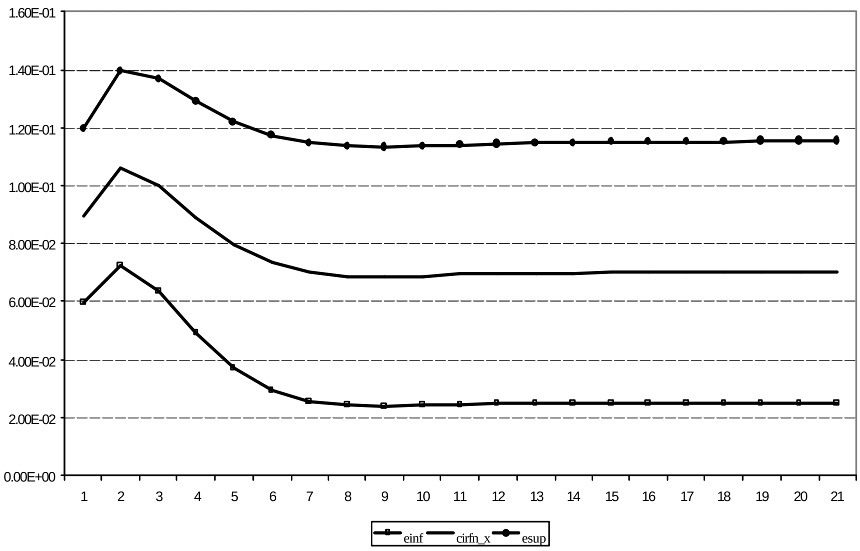


Figure 6.2.3:response of l to x

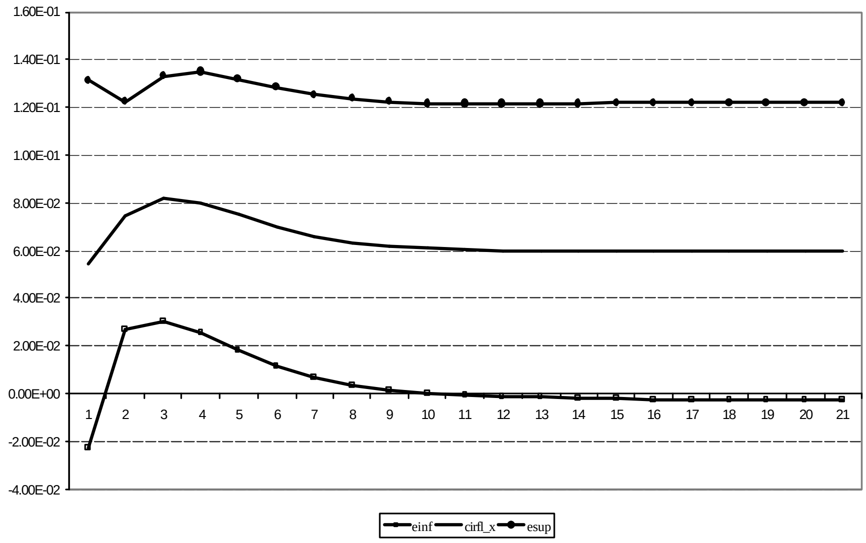


Figure 6.2.4:response of u to x

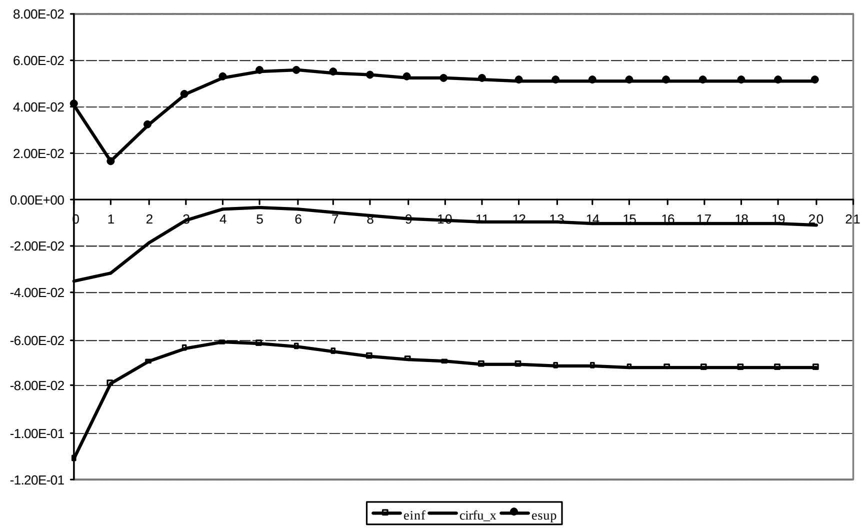


Figure 6.3.1:response of wr to mr

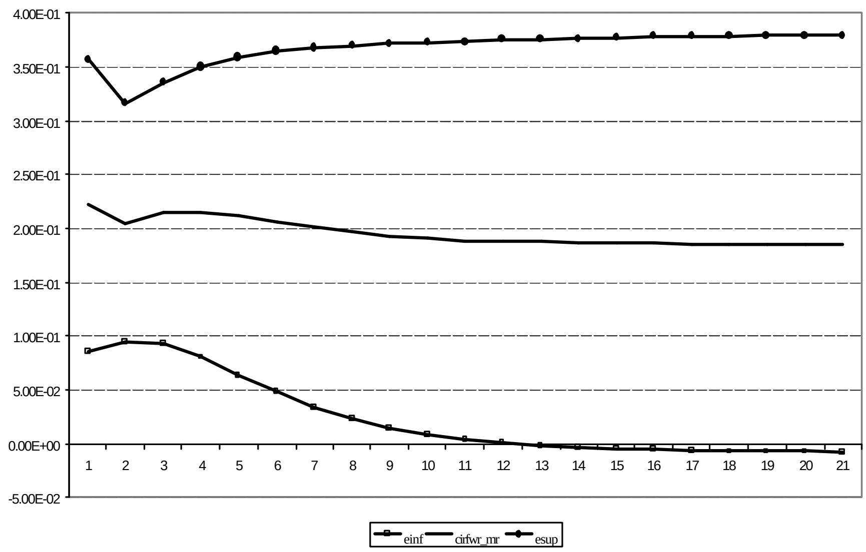


Figure 6.3.2:response of n to mr

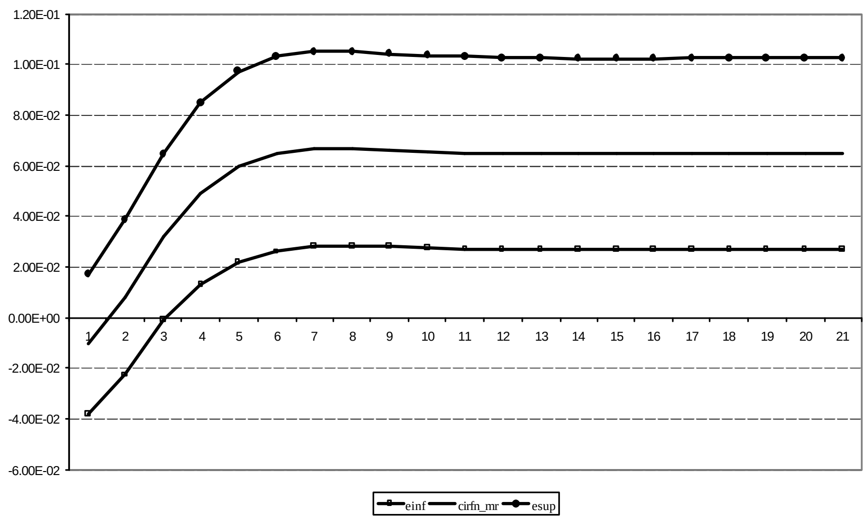


Figure 6.3.3:response of l to mr

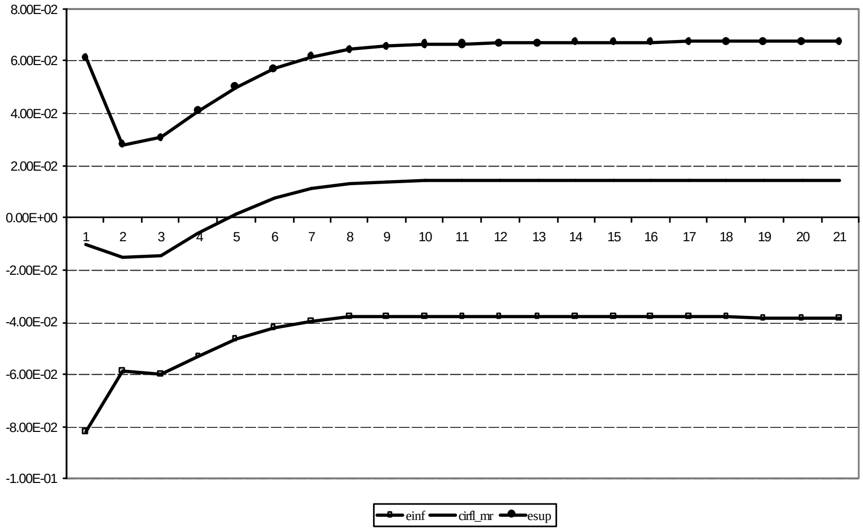


Figure 6.3.4:response of u to mr

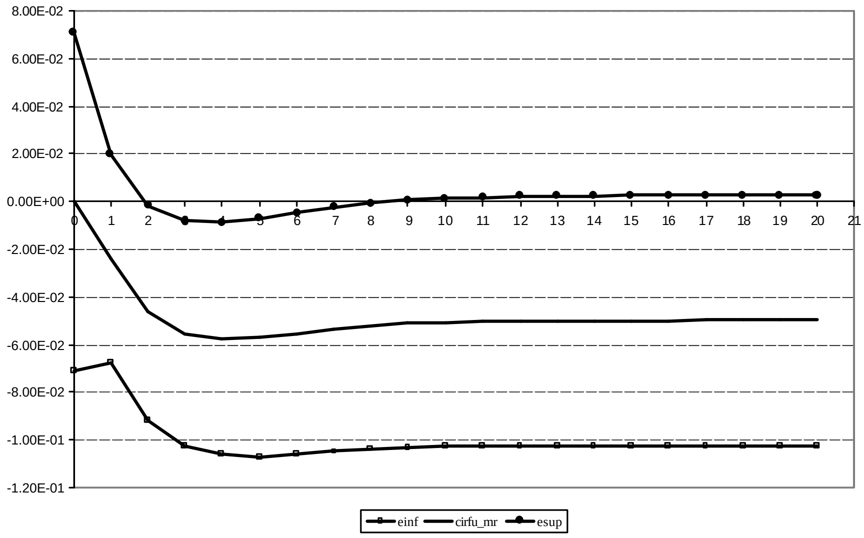


Figure 6.4.1:response of wr to ub

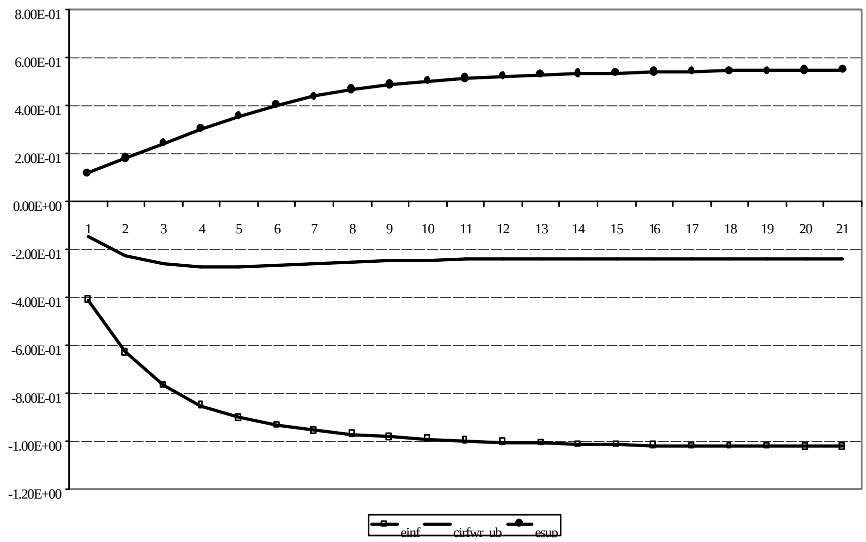


Figure 6.4.2:response of n to ub

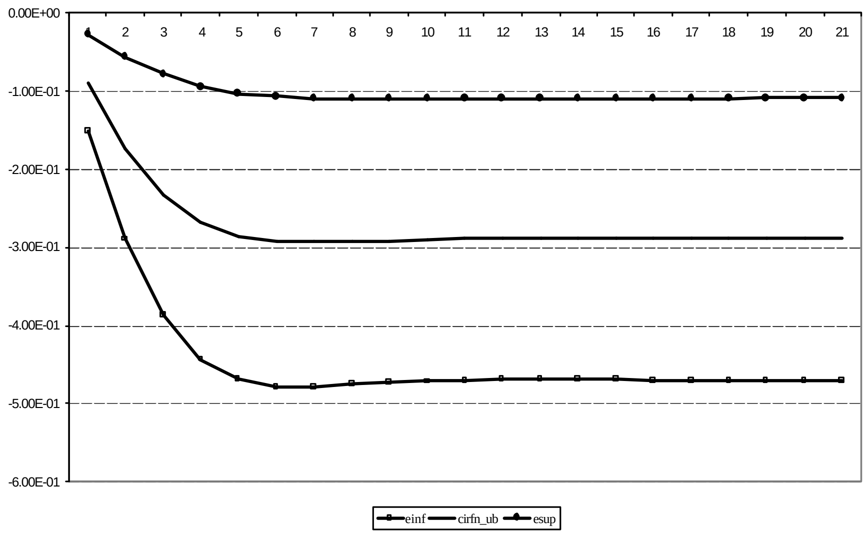


Figure 6.4.3:response of l to ub

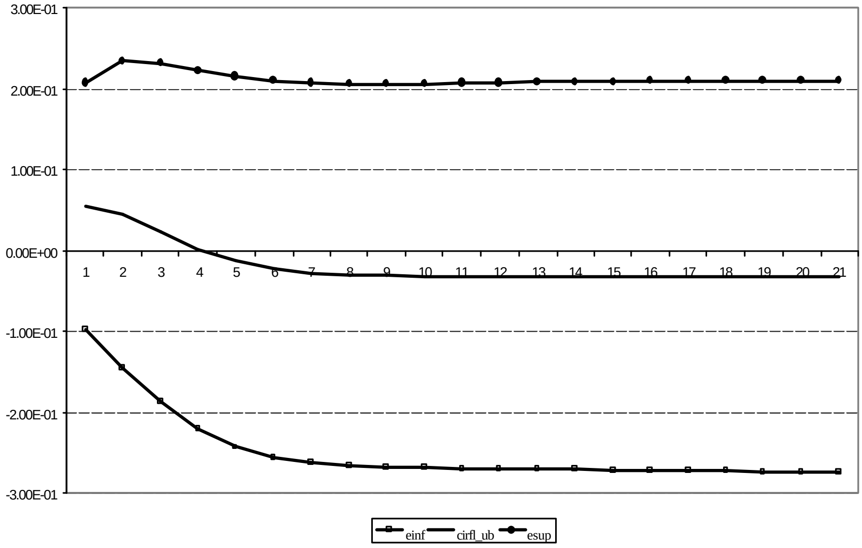


Figure 6.4.4:response of u to ub

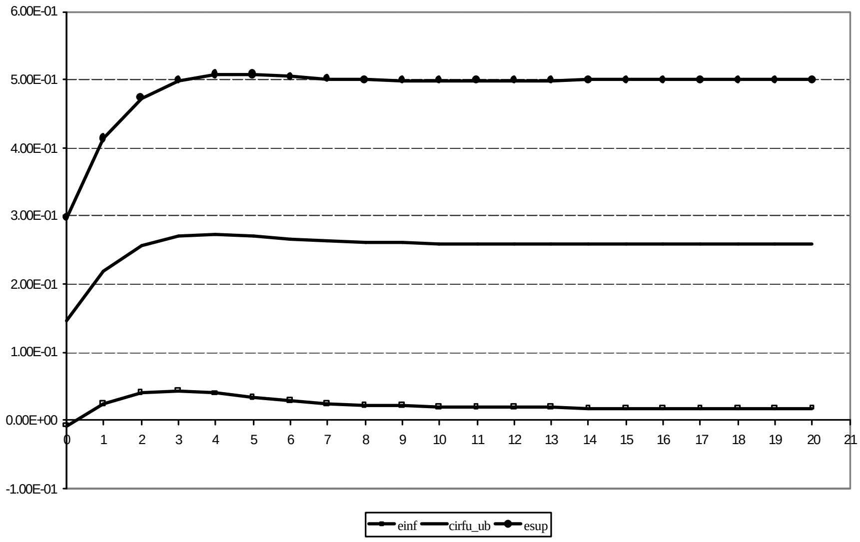


Figure 6.5.1:response of wr to lt

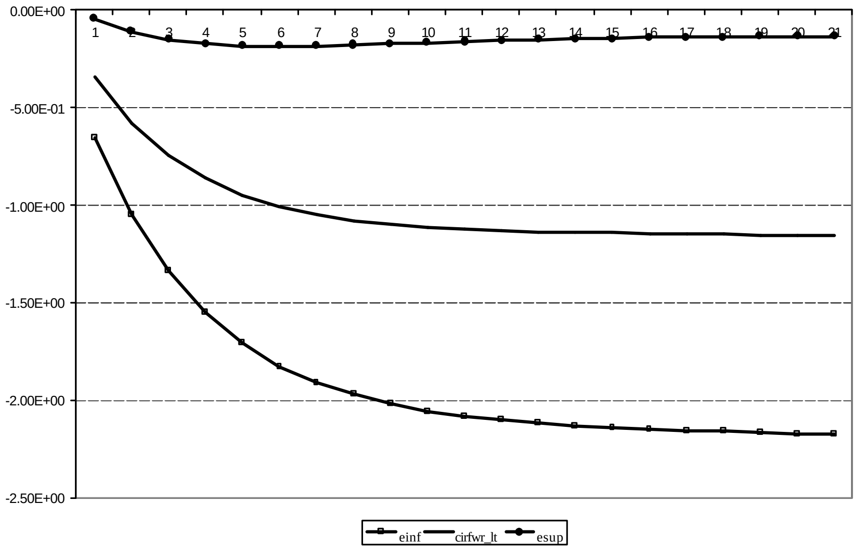


Figure 6.5.2:response of n to lt

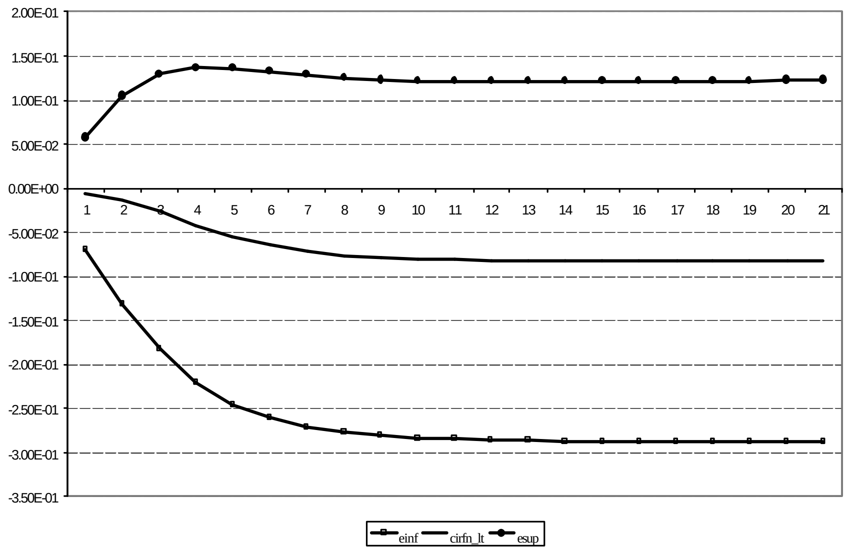


Figure 6.5.3:response of l to lt

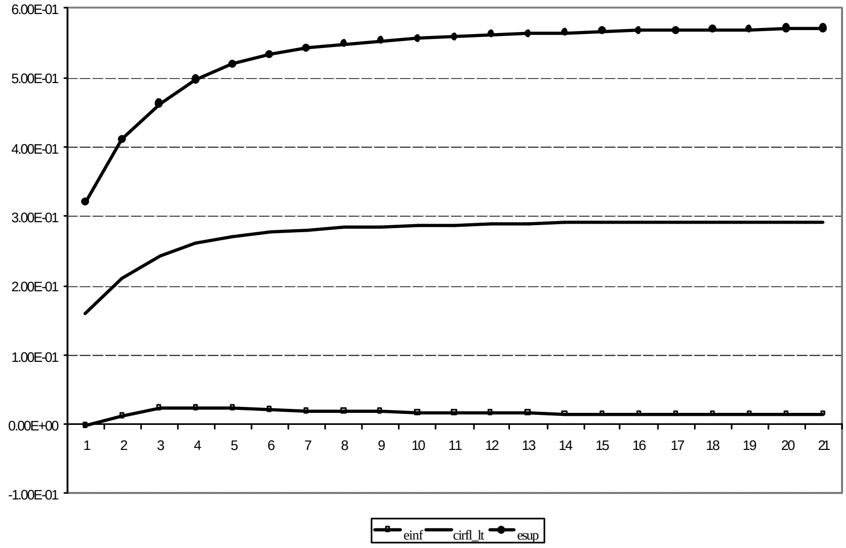


Figure 6.5.4:response of u to lt

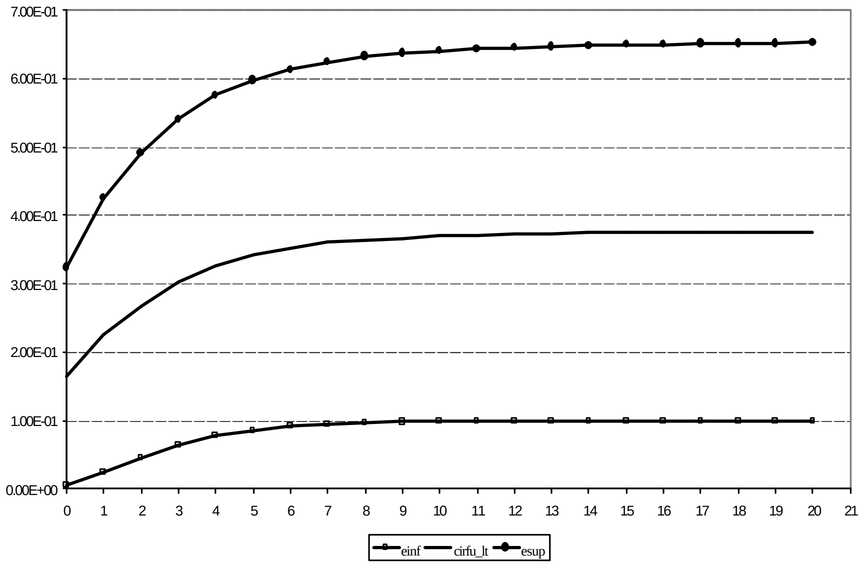


Figure 7.1: different configurations of w_t corresponding to different values of q_1, q_2, q_3

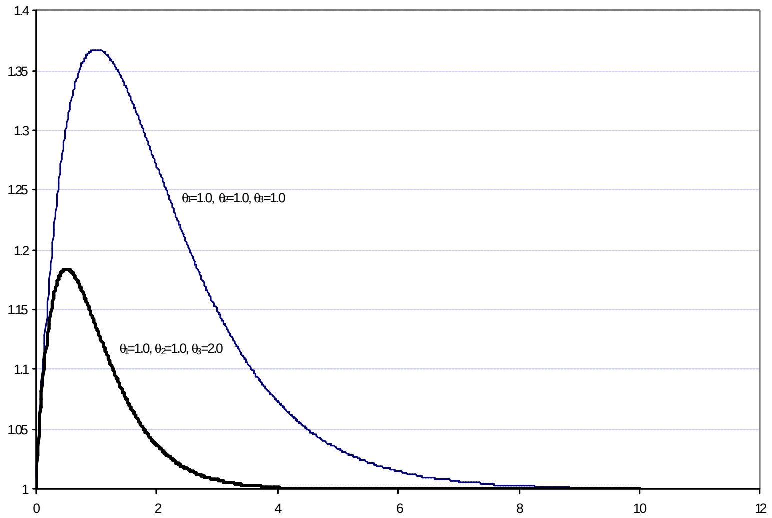
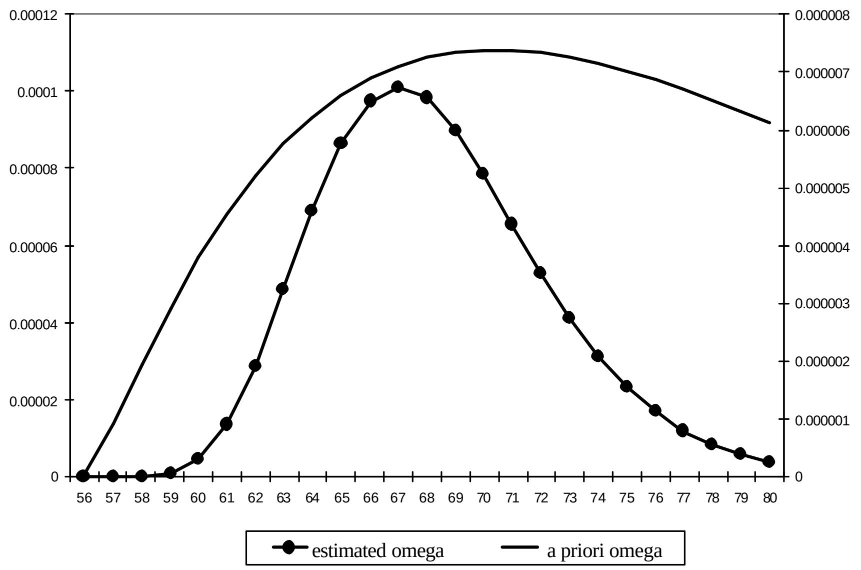


Figure 7.2: estimated and a priori DVI profiles



Appendix: data sources

In this appendix we describe the series used to measure the variables involved in our estimated models and their sources.
Real wages (*wr*): obtained as the ratio between the hourly wage rates (all industries) and the consumer price indexes (all items). **Source Datastream**.
Employment (*n*): total employment. **Source Datastream**.
Participation (*l*): total labour force. **Source Datastream**
Capital stock (*k*): quarterly series obtained on the basis of the annual series of capital stock. Intra-annual data reproduce the profile of the series on Gross Fixed Capital formation (private sector). **Source Datastream**.
Fiscal Policy Indicator (*x*): Government Investments. **Source Datastream.**
Monetary Policy Indicator (*mr*): obtained as a ratio between M2 and consumer price index (all items). **Source Datastream**
Unemployment Benefits (*UB*): average of the unemployment benefits Replacement Rates for two earnings levels, three family situations and three durations of unemployment (see [25]). The quartely data have been obtained by interpolation of the annual series **Source OECD.**
Labour taxes (*LT*): obtained as the ratio between the sum of income taxes, indirect taxes and Social Security Contributions (SSC both on workers and firms) and the sum of average gross wage and SSC on firms (see [25]). The quartely data have been obtained by interpolation of the annual series **Source OECD.**
All the variables are seasonally adjusted and expressed in real terms. In the estimation we used their logarithmic transformation.

References

[1] Amisano, G. and M. Serati (1999): "What goes up sometimes stays up: persistence of unemployment in different countries", *Discussion Paper n.9913*, University of Brescia.

[2] Balmaseda, M., J.J. Dolado, e D. Lopez-Salido (1997): "Hysteresis and the sources of shocks: a cross country analysis", *WP 9614, Centro de Estudios Monetarios Y Financieros*, Madrid, presented at the CEPR conference on "Unemployment persistence and the long-run: re-evaluating the natural rate", Vigo, 30th November- 1st December 1997.

[3] Bean, C. (1994), "European unemployment: a survey", *Journal of Economic Literature*, XXXII, 573-619.

[4] Bernardo, J.M., A.F. Smith (1994): "*Bayesian Theory*", Wiley, New York.

[5] Blanchard, O. (1997), "The medium run", *Brookings papers on Economic Activity*, 2, 89-158

[6] Blanchard, O. (1998), "Revisiting European unemployment. Unemployment, capital accumulation and factor prices", *NBER Working Paper*, n. 6656, May.

[7] Blanchard, O. (1999): "The role of shocks and institutions in the rise of European Unemployment", *Baffi Lecture* , Rome, January 1999.

[8] Blanchard, O. and L. Summers (1986), "Hysteresis and the European unemployment problem", in Stanley Fisher ed., *NBER Macroeconomics Annual*.

[9] Blanchard, O. and J.Wolfers (1999), "The role of shocks and institutions in the rise of European unemployment: the aggregate evidence", Harry Johnson lecture, *Economic Journal, 110,* March, *1-33*.

[10] Carter, C. K., and P. Kohn (1994): "On Gibbs sampling for state space models", *Biometrica*, 81, 541-553.

[11] Chib S., Greenberg E. (1995): "Hierarchical analysis of SUR models with extensions to correlated serial errors and time-varying parameter models", *Journal of Econometrics* (68)2 , 339-360.

[12] Christofides, C. A. (1996): "Italian unemployment 1975-1995: an analysis of macroeconomic shocks and policies using evidence from a Structural Vector Autoregression", *IMF Working Paper*, 96/36, April.

[13] Daveri F., Tabellini G. (2000), "Unemployment, growth and taxation in industrial countries", *Economic Policy: a European Forum*, April, Issue 30, 47-88

[14] Doan, T., R. Litterman, C. Sims (1984): "Forecasting and conditional projections using realistic prior distributions", *Econometric Reviews*, 3, 1-100.

[15] Geweke, J., (1999): "Using simulation methods for Bayesian econometric modelling: inference, development and communication", *Econometric Reviews*, 18, 1-74.

[16] Haffner, R., S. Nickell, G. Nicoletti, S. Scarpetta, G. Zoega (1999), "European integration, liberalisation and labour market performance", *Report for Rodolfo DeBenedetti Foundation.*

[17] Hamilton , J.(1994), "*Time series analysis*", Princeton University Press

[18] Harvey, A.C. (1993): "*Time Series Models*", Harvester Wheatsheaf, Hertfordshire.

[19] Ingram, B.F. and C.H. Witheman (1994): "Supplanting the 'Minnesota' prior : forecasting macroeconomic time series using real business cycle model priors", *Journal of Monetary Economics*, 34, 497-510.

[20] Kim, C.J., and C.R. Nelson (1999): *State space models with regime switching*, MIT Press, Cambridge, Mass.

[21] Layard R, Nickell S, Jackman R, (1991) "*Unemployment - Macroeconomic performance and the labour market*", Oxford, Oxford University Press.

[22] Ljungqvist, L. and T.J. Sargent (1998), "The European unemployment dilemma", *Journal of Political Economy*, 106, 514-551.

[23] Lindgren, G. (1978): "Markov regime models for mixed distributions and switching regressions", *Scandinavian Journal of Statistics*, 5, 81-91.

[24] Nickell S. (1997), "Unemployment and Labor Market rigidities: Europe versus North America", *Journal of Economic Perspectives*, 11, 3.

[25] Scarpetta, S. (1996): "Assessing the role of labour market policies and institutional settings on unemployment: a cross-country study", *OECD Economic Studies*, 26, 43-98.

Notes

[1] Paper presented at the Bank of Italy-CIDE Conference: *"Le basi quantitative della politica economica"*, 16th December 1999. We acknowledge precious suggestions from Fabio Bagliano, Monica Billio, Carlo Giannini and Domenico Sartore. We thank John Geweke for his Fortran routines library and Stefano Scarpetta for the OECD institutions variables dataset.

[2] amisano@eco.unibs.it. Dipartimento di Scienze Economiche, Università di Brescia
[3] mserati@liuc.it. Istituto di Economia, Università Cattaneo-LIUC di Castellanza

[4] All the variables are in logarithms.

[5] The number of wage pressures factors is very high and the same is true for their empirical and statistical measures. However, on the basis of the indications coming from the previous empirical literature on the relative importance of these factors and given the low quality of the available data, we limited our analysis to an unemployment benefit and a labour taxes measure.

[6] From expression (2.1) k_t-y_t=α(k_t-n_t), whereas (2.2) suggests that w_t-p_t=β(k_t-y_t). By substitution of the latter expression in the former, we have w_t-p_t= $\alpha\beta$(k_t-n_t), from which (2.4) can be directly obtained.

[7] ρ_ε, ρ_θ, ρ_υ are smaller than 1.

[8] For a more detailed description of the model and its solution see [1].

[9] Note that expectations on capital stock are defined in a different way with respect to the other ones. This happens because the capital stock is given at the beginning of each period, so that expectations on it coincide with its current value.

[10] Links with other formulae for the Kalman Filter are obtained by using the following matrix result:

$$\mathbf{D}= [\mathbf{ABA}'+\mathbf{C}]^{-1}=\mathbf{C}^{-1}\{\mathbf{I}_n-\mathbf{A}[\mathbf{B}^{-1}+\mathbf{A}'\mathbf{C}^{-1}\mathbf{A}]^{-1}\mathbf{A}'\mathbf{C}^{-1}\}$$

where **D** is ($n\times n$), **A** is ($n\times m$) and all the rest is conformable.

[11] Departure of Italy from EMS, revision of labour force definition, suppression of the "Scala Mobile".